Growth Forecasts, Belief Manipulation and Capital Markets

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Abstract

We analyze how a benevolent, privately informed government agency would optimally release information about the economy’s growth rate when the agents hold heterogeneous beliefs. We model two types of agent: “conforming” and “dissenting.” The former has a prior that is identical to that of the government agency, whereas the latter has a prior that differs from that of the government agency. We identify both informative and uninformative equilibria and demonstrate that the uninformative equilibria can dominate the informative ones in terms of ex-post social welfare.

Keywords: Social welfare, information, forecasting, asset pricing, heterogeneous beliefs

JEL codes: D83, G11, G12

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1 Introduction

There is reason to believe that governments are, in most cases, better informed about the future of the macroeconomy than the vast majority of private investors. Publicly announced macroeconomic forecasts from official bodies with ties to the government may therefore contain valuable information about the future state of the economy. That is, if the government would like to reveal this information to the public. In addition to political motives for producing optimistic forecasts, one can easily think of cases where the government would not want to announce a bad but informative forecast in order to avoid widespread panics and liquidity shortages in the financial markets. This paper analyzes how a government agency would optimally choose to release information about future growth rates.

Although there is generally a bias towards optimism, the empirical evidence on the informativeness of government agencies’ growth forecasts is mixed. Jonung and Larch (2006) find a significant upward bias in government forecasts of both real and potential GDP growth in Germany, Italy and France. In addition, they find that, while the official GDP forecasts of France, Germany and the UK outperform a naïve forecast (latest value), those of Italy do not. That is, in the case of Italy, one would have been more accurate in one’s predictions by using the latest value for GDP growth as a forecast instead of the official forecast from the Italian government. Similarly, Ashiya (2007) finds a significant upward bias in the official government forecasts of real GDP growth in Japan and that they are inferior to using an average forecast of private institutions or a vector autoregression...
forecast based on real-time data. The findings in Tsuchiya (2013) cast further doubt on the usefulness of the Japanese government forecasts.

From a theoretical viewpoint, we analyze how a privately informed, benevolent government agency would choose to release information regarding future growth rates and its effects on asset prices and portfolio holdings.\footnote{The government agency’s problem in this paper can be viewed as a constrained social planner’s problem with a constraint on the amount of information available to the planner, as in Angeletos and La’O (2011).} We consider agents different in their prior beliefs; some (which we call \textit{conforming} agents) trust the government agency in that they share its prior beliefs, whereas others (which we call \textit{dissenting} agents) do not and have a prior that differs from that of the government agency—for instance, because of previous detection of lies.

We identify both informative and uninformative equilibria and we find that, when the mass of the dissenting agents is sufficiently high or when the difference in priors is sufficiently large, the uninformative equilibria dominate the informative ones in terms of social welfare. In an uninformative equilibrium, the information sent by the government agency is disregarded, and agents invest on their own as in any cheap-talk equilibrium. In contrast, in an informative equilibrium, agents act on the forecast provided by the government agency.

When the dissenting agents have a more pessimistic (optimistic) prior regarding dividend growth than the conforming agents, we find that a larger the mass of dissenting agents leads to a higher (lower) bond price, or, equivalently, a lower (higher)
rate. The point is that if the dissenting agents have a more pessimistic (optimistic) prior regarding dividend growth than the conforming agents, then the bond appears more (less) attractive to the dissenting agents.

The paper is related to the literature on the social value of public information. Drèze (1960) and Hirshleifer (1971) identify the possibility that information may have a negative social value, which became known as the “Hirshleifer effect.” Since then, several authors have investigated the robustness of this result (e.g., Marshall, 1974; Ng, 1975; Green, 1981; Hakansson, Kunkel, and Ohlson, 1982; Schlee, 2001; Campbell, 2004). In the context of a “beauty contest” à la Keynes (1936), Morris and Shin (2002) show that greater provision of public information may not improve welfare. This result has gained attention in the media (Economist, 2004) and spurred academic debate (Svensson, 2006; Morris, Shin, and Tong, 2006). We contribute to this literature mainly by showing how the presence of heterogeneity among the agents can induce both informative and uninformative equilibria, depending on the relative masses of the agents. Albornoz, Esteban, and Vanin (2012) also consider a related problem of government disclosure of economic information, but with economic distortions such as taxes or monopolies. They find that those distortions prevent truthful equilibrium reports whenever they are too strong; in contrast, we show that heterogeneity of agents’ beliefs alone, and without distortions, triggers optimal misreporting.

Another branch of the literature to which we contribute is that of cheap-talk equilibria (Crawford and Sobel, 1982; Farrell and Gibbons, 1989; Stein, 1989; Matthews, Okono-Fujiwara, and Postlewaite, 1991; Admati and Pfleiderer, 2004; Kawamura, 2011, among
others). In their seminal article, Crawford and Sobel (1982) explicitly model the choice to manipulate, allowing it to be endogenously optimally chosen. They consider a better-informed sender who sends a signal to a receiver who then takes an action that affects the welfare of both. A central result is that “the more similar agents’ preferences, the more informative the equilibrium signal” (Crawford and Sobel, 1982, p. 1432). Our result that the uninformative equilibria can dominate the informative ones in terms of social welfare is in line with this finding: Our numerical results suggest that the mass of dissenting agents or the difference in priors needs to be sufficiently large for the uninformative equilibria to dominate and, further, we notice that, as the mass of the dissenting agents or the difference in priors becomes larger, the government agency’s objective becomes more different from that of the median agent.

For simplicity, we consider what is effectively a one-period model, but our results readily extend to more general settings (e.g., repeated interaction under IID growth rates and “stationary” updating rules, as discussed in the appendix). Further, we consider rational Bayesian updaters, and the heterogeneity in beliefs stems from differences in priors. The model nonetheless extends to other behavioral assumptions on the agents’ updating processes—for instance, when one group of agents purposely ignores any signal, whereas another group of agents consists of rational Bayesian updaters.

The remainder of the paper is organized as follows. In Section 2, we introduce our model, and in Section 3, we present our theoretical results. Finally, the fourth section concludes the paper. All the proofs are in the appendix.
2 Model

We consider what is effectively a one-period exchange-only Lucas (1978) economy with three dates \( t = 1, 2, 3 \), as explained below.\(^2\) In this economy, there is a continuum of atomless agents with a total mass of one, who are *price takers*, and a government agency. There is one consumption good and one asset in positive net supply (the risky tree) and a full set of contingent claims in zero net supply, from which a risk-free asset (bond) can be constructed.\(^3\) There are two states (high \( h \) and low \( l \)); hence, it suffices to have two assets with linearly independent payoffs (e.g., the risky tree and a bond) to complete the market. The bond yields one unit of the consumption good at the last date \( t = 3 \), while the payoff of the risky tree (stock) at \( t = 3 \) depends on one of the two realizations of nature (high \( h \), or low \( l \)) as follows. The stock returns a known dividend, \( D_2 \), at date \( t = 2 \) and a second dividend, \( \tilde{D}_3 = D_2 g \), at the final date, \( t = 3 \), where, for simplicity, the growth rate \( g \) is a binary random variable, described below. Without loss of generality, we assume that the ownership of the risky tree is initially uniformly distributed across the agents; this ownership is their sole endowment.

According to the government agency’s prior beliefs, the growth rate is *high* (\( g = g_h \)).

\(^2\)In the appendix, we explain how these results extend to the case when there are several periods (see the section “Two(n)-period case”).

\(^3\)Note that all assets except the risky tree are in zero net supply.
with probability \( p_h \) and low \( g = g_l \) with probability \( 1 - p_h \). That is,

\[
g = \begin{cases} 
  g_h & \text{with probability } p_h \\
  g_l & \text{with probability } 1 - p_h, 
\end{cases} 
\]

where \( p_h \in (0, 1) \) and \( 0 < g_l < g_h \).

At date \( t = 1 \), the government agency has private information regarding the realization of the random variable \( g \). This private information takes the form of a partially revealing signal, \( s_R \). The government agency’s private signal returns the correct growth rate with probability \( \xi \in \left( \frac{1}{2}, 1 \right) \) and an incorrect growth rate with probability \( 1 - \xi \). We say that the signal \( s_R \) partially reveals the growth rate because the distribution of \( s_R \) conditional on the growth rate \textit{depends} on the growth rate. Formally, we have

\[
s_R = \begin{cases} 
  g & \text{with probability } \xi \\
  \neg g & \text{with probability } 1 - \xi, 
\end{cases} 
\]

where \( \xi \in \left( \frac{1}{2}, 1 \right) \). In the appendix, we discuss the extreme case when the signal \( s_R \) gives the true growth rate with probability one (\( \xi = 1 \)).

A group of conforming agents with positive mass has a prior regarding the future growth rate that is identical to the government agency’s prior. In addition, there is a group of dissenting agents of mass \( v \in (0, 1) \), whose prior beliefs deviate from the government agency’s prior. Their initial probability of a high growth rate is \( p_{h}^{d} \), where \( p_{h}^{d} \in (0, 1) \) and \( p_{h}^{d} \neq p_h \). We let \( \Pr(\cdot) \) denote the conforming agents' probability operator, and we let \( \Pr^{d}(\cdot) \) denote the dissenting agents’ probability operator.

\(^4\)Here, “\( \neg g \)” means “not \( g \).” That is, if \( g = g_h \), then \( \neg g = g_l \) and if \( g = g_l \), then \( \neg g = g_h \).
At \( t = 1 \), after receiving its private signal, the government agency decides on the probability by which it forwards its private signal \( s_R \). That is, it sends a public signal \( s \), where \( s = g_h \) or \( s = g_l \), so that \( s = s_R \) or \( s = \neg s_R \).

The agency maximizes *ex-post* social welfare,\(^5\) as discussed below, by choosing the probability of forwarding its private signal to the agents. In the terminology of Angeletos and La’O (2011), the government agency solves a constrained social planner’s problem with a constraint on the amount of information available to the planner. At date \( t = 1 \), the agents assign conditional probabilities \( \hat{\theta}_h \) and \( \hat{\theta}_l \) to the possibility that the government forwards its private signal, conditional on it being \( s_R = g_h \) and \( s_R = g_l \), respectively.

At \( t = 2 \), the agents receive the government agency’s previously chosen signal and they update their beliefs about \( g \) as a function of the received signal, as well as the probability that the government agency forwards its private signal, in a Bayesian manner. According to their newly formed beliefs, at date \( t = 2 \), they allocate their current wealth between current consumption and the two assets yielding consumption at \( t = 3 \).

Every agent is a standard von Neumann–Morgenstern expected-utility maximizer, as of \( t = 2 \), with an intertemporal discount factor \( \beta \in (0, 1) \); that is, agents seek to maximize

\[
u(C^i_2) + \beta E^{\tilde{F}^i} \left[ u \left( \tilde{C}^i_3 \right) \bigg| s \right] i \in [0, 1], \tag{3}\]

where \( C^i_2 \) denotes agent \( i \)’s initial consumption,\(^6\) \( \tilde{C}^i_3 \) denotes his final state-contingent

\(^5\) Here, *ex-post* refers to the circumstance that the government agency considers the expected utility of the agents *conditional on its own received signal*.

\(^6\) That is, in order to start with a generally applicable notation, we use \( i \) to index agent \( i \). However, in this paper, we consider two *types* of agents.
consumption, and $\hat{P}\imath$ is his probability measure. The elementary utility function $u$ is concave, strictly increasing, and twice-continuously differentiable. In addition, we assume that $u'(c) \to +\infty$ as $c \to 0$. The agents’ budget constraints are standard.

The government agency’s objective is to maximize social welfare from $t = 1$ by choosing the appropriate signal to send while taking as given the individual demand functions and prices for which markets clear. In particular, the strategy for the agency is to choose a vector of probabilities, $\theta = (\theta_\imath, \theta_\lambda)$, of forwarding its private signal, where $\theta_\imath$ is the probability of forwarding the private signal, conditional on it being $s_R = g_\imath$, and, correspondingly, $\theta_\lambda$ is the probability of forwarding the private signal, conditional on it being $s_R = g_\lambda$. Thus, the agency seeks to maximize the expressions

$$W_j = E^{P} \left[ \int_{[0,1]} \left( u(C_i^j) + \beta u\left( \tilde{C}_i^3 \right) \right) \, di \mid s_R = g_j \right], \ j = \imath, \lambda \quad (4)$$

over $\theta_\imath$ and $\theta_\lambda$ (where $0 \leq \theta_\imath \leq 1$ and $0 \leq \theta_\lambda \leq 1$). Here, $P$ denotes the agency’s probability measure (under which its strategy $\theta$ is known), and $C_i^j$ and $\tilde{C}_i^3$ are the equilibrium consumption levels such that markets clear.

The structure of the signals, $g_\imath$, $g_\lambda$, $\xi$, the agents’ preferences and beliefs, and the government agency’s objective function are common knowledge. Formally, we consider the following equilibrium concept.$^7$

**Definition 1.** An equilibrium for this economy is a set of consumption and investment decisions $\{C_{2i}^\imath, \phi_{1i}^\imath\}_{i \in [0,1]}$, a government agency strategy $(\theta^* = (\theta_\imath^*, \theta_\lambda^*))$, a set of prices

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$^7$Note that $\theta^*$ (the vector of probabilities by which the government agency forwards its private signal) determines its equilibrium signal.
$(S, B)$, and a system of beliefs $(\widehat{\Pr}(g = g_h \mid s)$ and $\widehat{\Pr}_d (g = g_h \mid s))$ such that

i) $\{C_{s,i}^*, \phi_{s,i}^*\}_{i \in [0,1]}$ solve the agents’ consumption and investment problems given $\widehat{\Pr}(g = g_h \mid s)$, $\widehat{\Pr}_d (g = g_h \mid s)$, $S$ and $B$;

ii) $\theta^*$ solves the government agency’s problem given $\widehat{\Pr}(g = g_h \mid s)$ and $\widehat{\Pr}_d (g = g_h \mid s)$;

iii) Markets clear, $\int_0^1 C_{s,i}^* \, di = D_2$ and $\int_0^1 \phi_{s,i}^* \, di = 1$; and

iv) $\widehat{\Pr}(g = g_h \mid s)$ and $\widehat{\Pr}_d (g = g_h \mid s)$ are computed according to Bayes’ rule whenever possible. If $\widehat{\theta}_h = 1$ and $\widehat{\theta}_l = 0$, we let $\widehat{\Pr}(g = g_h \mid s = g_l) = \frac{p_h(1-\xi)}{p_h(1-\xi) + (1-p_h)\xi}$ and $\widehat{\Pr}_d (g = g_h \mid s = g_l) = \frac{p_h\xi}{p_h\xi + (1-p_h)(1-\xi)}$. If $\widehat{\theta}_h = 0$ and $\widehat{\theta}_l = 1$, we let $\widehat{\Pr}(g = g_h \mid s = g_l) = \frac{p_d(1-\xi)}{p_d(1-\xi) + (1-p_d)\xi}$ and $\widehat{\Pr}_d (g = g_h \mid s = g_l) = \frac{p_d\xi}{p_d\xi + (1-p_d)(1-\xi)}$.

v) Beliefs are consistent with the government agency’s set of optimal policies—i.e.,

$\widehat{\theta} \in \Theta^*$, where $\Theta^* = \{\theta^* \mid \theta^*$ solves the government agency’s problem given $\widehat{\Pr}(g = g_h \mid s)$ and $\widehat{\Pr}_d (g = g_h \mid s)\}$

Notice that in iv, we also specify posterior beliefs “off the equilibrium path,” when Bayes’ rule cannot be used. In the case when $\widehat{\theta}_h = 1$ and $\widehat{\theta}_l = 0$, observing the public signal $s = g_l$ is a zero-probability event. After observing $s = g_l$, the agents must realize that something must be wrong about their beliefs: it cannot be that both $\widehat{\theta}_h = 1$ and $\widehat{\theta}_l = 0$. In this case, we imagine that the agents still believe that, if the government agency receives $s_R = g_h$, it will forward this private signal with probability one; however, they revise their belief regarding the event that the government agency would forward $s_R = g_l$. That is, they assign a nonzero probability to it. Similarly, in the case when $\widehat{\theta}_h = 0$ and
\( \hat{\theta}_1 = 1 \), observing \( s = g_h \) is a zero-probability event. In analogy to the previous case, we imagine that, upon observing \( s = g_h \), the agents still think that if the government agency receives \( s_R = g_l \), it forwards this signal with probability one; however, they revise their belief regarding the event that the government agency would forward \( s_R = g_h \). These types of revisions result in the conditional probabilities that we state in item iv of the above definition (see the next section for derivations).

\section{Results}

In this section, we present our results. First, we consider the effect of heterogeneity on investment decisions and asset prices. Thereafter, we consider the effect on the government agency’s decision, showing that the uninformative equilibria we identify can in fact dominate the informative equilibria from the point of view of social welfare.

Since \( g_h \neq g_l \), markets are complete, and we can solve for the equilibrium using two Arrow–Debreu (AD) securities: The first AD security delivers one unit of consumption if the growth rate turns out to be high \( (g = g_h) \) and zero units otherwise while the second AD security delivers one unit of consumption if the growth rate turns out to be low \( (g = g_l) \) and zero units otherwise. The prices of these AD securities generically depend on the government agency’s public signal. That is, the government agency can affect prices through its signaling. We denote the price of the first AD security by \( q_{hj} \) and the price of the second by \( q_{lj} \), where the index \( j \) indicates the realization of the signal, where \( j = h \) in case of a high-growth signal \( (s = g_h) \) and \( j = l \) in case of a low-growth signal \( (s = g_l) \).
The agents’ posterior beliefs are of crucial importance. Item iv in the definition of the equilibrium (Definition 1) states that these should be computed according to Bayes’ rule whenever possible and specifies beliefs in cases when it is not possible to use Bayes’ rule. Proposition 1 determines posterior beliefs in cases when it is possible to apply Bayes’ rule.

**Proposition 1.** According to Bayes’ rule, the agents’ posterior beliefs regarding the growth rate are given by

\[
\hat{\mu}_{hh}^d \equiv \Pr^d(g = g_h | s = g_h) = \frac{p_h \xi \hat{\theta}_h + p_h^d (1 - \xi) \left(1 - \hat{\theta}_l\right)}{(p_h \xi + (1 - p_h^d) (1 - \xi)) \hat{\theta}_h + ((1 - p_h^d) \xi + p_h^d (1 - \xi)) \left(1 - \hat{\theta}_l\right)}; \tag{5}
\]

\[
\hat{\mu}_{hl}^d \equiv \Pr^d(g = g_h | s = g_l) = \frac{p_h \xi \left(1 - \hat{\theta}_h\right) + p_h^d (1 - \xi) \hat{\theta}_l}{(p_h \xi + (1 - p_h^d) (1 - \xi)) \left(1 - \hat{\theta}_h\right) + ((1 - p_h^d) \xi + p_h^d (1 - \xi)) \hat{\theta}_l}; \tag{6}
\]

\[
\hat{\mu}_{hh} \equiv \Pr(g = g_h | s = g_h) = \frac{p_h \xi \hat{\theta}_h + p_h (1 - \xi) \left(1 - \hat{\theta}_l\right)}{(p_h \xi + (1 - p_h) (1 - \xi)) \hat{\theta}_h + ((1 - p_h) \xi + p_h (1 - \xi)) \left(1 - \hat{\theta}_l\right)}; \tag{7}
\]

\[
\hat{\mu}_{hl} \equiv \Pr(g = g_h | s = g_l) = \frac{p_h \xi \left(1 - \hat{\theta}_h\right) + p_h (1 - \xi) \hat{\theta}_l}{(p_h \xi + (1 - p_h) (1 - \xi)) \left(1 - \hat{\theta}_h\right) + ((1 - p_h) \xi + p_h (1 - \xi)) \hat{\theta}_l}. \tag{8}
\]

If \(\hat{\theta}_h = 1\) and \(\hat{\theta}_l = 0\), then observing \(s = g_l\) is a zero-probability event for both types of agent and, similarly, if \(\hat{\theta}_h = 0\) and \(\hat{\theta}_l = 1\), then observing \(s = g_h\) is a zero-probability event for both types of agent. In these cases, Bayes’ rule cannot be used, but we still want to specify updated probabilities “off the equilibrium path” where the agents would observe \(s = g_l\) and \(s = g_h\), respectively. This is done in item iv of the definition of the equilibrium (Definition 1). The discussion below Definition 1 provides the intuition behind the conditional probabilities that are specified in item iv of the definition.

In order to achieve tractability, we solely focus on logarithmic utility functions, but, as we demonstrate in the appendix, our results extend to nonlogarithmic CRRA utility.
functions. From the calculations in the appendix, we also find that, in equilibrium, the ratios of state-contingent consumption equal the ratios of beliefs,

\[
\frac{C_{3hj}}{C_{3d}} = \frac{\hat{\mu}_{hj}}{\hat{\mu}_{hj}} \\
\frac{C_{3lj}}{C_{3d}} = \frac{(1 - \hat{\mu}_{hj})}{(1 - \hat{\mu}_{hj})},
\]

meaning that, if the conforming agents believe that a state is more likely than the dissenting agents do, they will also consume more in that state than the dissenting agents.\(^8\)

Since the conforming and the dissenting agents only differ in terms of their priors, one group will always be more optimistic (namely, the one with the highest prior probability of a high growth rate). So if, for example, the conforming agents have a higher prior probability of a high growth rate, then the members of this group will always consume more in the high-growth state than the dissenting agents, regardless of the government agency’s public signal. However, the magnitude of the difference is generically affected by the government agency’s public signal.

The state-contingent consumptions of conforming and dissenting agents can be reached by trading the stock and the bond. Focusing on the conforming agents, we find that their optimal number of stocks is given by\(^9\)

\[
\phi_j = \frac{\hat{\mu}_{hj}(1 - \hat{\mu}_{hj})(g_h - g_l) + v(\hat{\mu}_{hj} - \hat{\mu}_{hj})(1 - \hat{\mu}_{hj})g_h + \hat{\mu}_{hj}g_h}{(g_h - g_l)(v\hat{\mu}_{hj} + (1 - v)\hat{\mu}_{hj})v(1 - \hat{\mu}_{hj}) + (1 - v)(1 - \hat{\mu}_{hj})},
\]

---

\(^8\)Note that they start out with identical endowments.

\(^9\)We can solve for their optimal asset holdings from \(\phi_j \cdot D_2 g_h + \eta_j \cdot 1 = C_{3hj}\) and \(\phi_j \cdot D_2 g_l + \eta_j \cdot 1 = C_{3ij}\), where \(\phi_j\) is the number of stocks and \(\eta_j\) is the number of bonds held by a conforming agent.
and their optimal number of bonds is given by
\[
\eta_j = \frac{D_2g_hg_iv(\hat{\mu}_{hj}^d - \hat{\mu}_{hj})}{(g_h - g_i)(v\hat{\mu}_{hj}^d + (1 - v)\hat{\mu}_{hj})\left(v(1 - \hat{\mu}_{hj}^d) + (1 - v)(1 - \hat{\mu}_{hj})\right)}.
\]  \hspace{1cm} (12)

Here, we can see that, provided the conforming agents have a higher prior probability of a high growth rate than the dissenting agents, they will short sell the bond in order to invest more in the stock; conversely, if they have a lower prior probability, they will have a long position in the bond. The government agency’s public signal will then generically affect magnitudes.

### 3.1 Optimal asset holdings and dissenting agents

We now analyze how the mass of dissenting agents impacts the overall distribution of bond and stock holdings. Taking the derivative of the conforming agents’ optimal bond holding with respect to \(v\), we have
\[
\frac{\partial \eta_j}{\partial v} = \frac{D_2g_hg_i(\hat{\mu}_{hj}^d - \hat{\mu}_{hj})\left((1 - \hat{\mu}_{hj})\hat{\mu}_{hj} + (\hat{\mu}_{hj} - \hat{\mu}_{hj}^d)^2 v^2\right)}{(g_h - g_i)(v\hat{\mu}_{hj}^d + (1 - v)\hat{\mu}_{hj})^2\left(v(1 - \hat{\mu}_{hj}^d) + (1 - v)(1 - \hat{\mu}_{hj})\right)^2}.
\]  \hspace{1cm} (13)

Hence, the sign of the derivative depends on the sign of \(\hat{\mu}_{hj}^d - \hat{\mu}_{hj}\), meaning that, if the dissenting agents have a more pessimistic (optimistic) prior, then an increase in the mass of dissenting agents will lead the conforming agents to decrease (increase) their bond holdings. From (12), we know that, in the case when the dissenting agents have a more pessimistic prior, the bond holdings of the conforming agents are negative; whereas, in the case when the dissenting agents have a more optimistic prior, the bond holdings of the conforming agents are positive. Thus, with respect to bond holdings, the mass of dissenting agents will only affect magnitudes, not signs.
We can also study how the mass of the dissenting agents affects the conforming agents’ optimal stock holding:

\[
\frac{\partial \phi_j}{\partial v} = \frac{\hat{\mu}_{hj} - \hat{\mu}^d_{hj}}{g_h - g_l} \left( \frac{\hat{\mu}_{hj} g_h}{v \hat{\mu}^d_{hj} + (1 - v)\hat{\mu}_{hj}} \right)^2 + \frac{(1 - \hat{\mu}_{hj}) g_h}{v (1 - \hat{\mu}^d_{hj}) + (1 - v)(1 - \hat{\mu}_{hj})}.
\] (14)

That is, the sign of this derivative depends on the sign of \(\hat{\mu}_{hj} - \hat{\mu}^d_{hj}\). This is opposite the result we obtained regarding the conforming agents’ bond holdings. Thus, if the dissenting agents have a more pessimistic (optimistic) prior, then the effect of an increase in the mass of the dissenting agents will increase (decrease) the conforming agents’ stock holdings. Here, the mass of the dissenting agents can affect signs, as can be seen in (11).

We next notice that the government’s signal does not affect stock prices, while it directly affects holdings.

**Lemma 1.** In equilibrium, the stock price is independent of individual beliefs.

This result is specific to the log utility case. For risk aversions different from one, agents’ beliefs, and thus the government agency’s signal, will generically affect the stock price. However, the bond price will generically depend on beliefs also under log utility.

We can consider the effect of a larger mass of dissenting agents on the bond price:

\[
\frac{\partial B_j}{\partial v} = \beta \left( \frac{1}{g_h} - \frac{1}{\hat{g}_h} \right) (\hat{\mu}_{hj} - \hat{\mu}^d_{hj}).
\] (15)

If the dissenting agents have a more pessimistic (optimistic) prior regarding dividend growth than the conforming agents, then the bond is more (less) attractive to the dissenting agents. Thus, the larger (smaller) the mass of dissenting agents, the higher (lower) the bond price, or, equivalently, the lower (higher) the interest rate.
3.2 Government’s reaction

We now consider the government agency’s problem in this economy. The agency seeks to maximize ex-post social welfare by choosing the probability by which it forwards its revealing private signal. The agency’s objective function is given in (4).\(^{10}\) With two types of log-utility agents (conforming and dissenting) of mass \(v\) and \(1 - v\), respectively, we can write the agency’s objective function as

\[
W_j = \theta_j \left\{ v \left( \ln C_{2} + \beta \left[ \hat{\mu}_{bj}^{G} \ln C_{3hj} + (1 - \hat{\mu}_{bj}^{G}) \ln C_{3lj} \right] \right) \\
+ (1 - v) \left( \ln C_{2} + \beta \left[ \hat{\mu}_{bj}^{G} \ln C_{3hj} + (1 - \hat{\mu}_{bj}^{G}) \ln C_{3lj} \right] \right) \\
+ (1 - \theta_j) \left\{ v \left( \ln C_{2} + \beta \left[ \hat{\mu}_{bj}^{G} \ln C_{3h\neg j} + (1 - \hat{\mu}_{bj}^{G}) \ln C_{3l\neg j} \right] \right) \\
+ (1 - v) \left( \ln C_{2} + \beta \left[ \hat{\mu}_{bj}^{G} \ln C_{3h\neg j} + (1 - \hat{\mu}_{bj}^{G}) \ln C_{3l\neg j} \right] \right) \right\}, \tag{16}
\]

where the index \(j\) indicates whether the government agency has observed \(s_R = g_h\) \((j = h)\) or \(s_R = g_l\) \((j = l)\), \(\neg j\) stands for “not \(j\),” so that \(\neg h = l\) and \(\neg l = h\), and \(\hat{\mu}_{bj}^{G}\) is the government agency’s posterior probability, after having observed \(s_R = g_j\). Using Bayes’ rule, we can calculate the agency’s posteriors as

\[
\hat{\mu}_{hh} = \frac{p_{h} \xi}{p_{h} \xi + (1 - p_{h})(1 - \xi)}, \tag{17}
\]

\[
\hat{\mu}_{hl} = \frac{p_{h}(1 - \xi)}{p_{h}(1 - \xi) + (1 - p_{h})\xi}. \tag{18}
\]

\(^{10}\)In the appendix, we show that similar results are obtained when considering an alternative ex-ante welfare function, according to which the government agency maximizes the weighted sum of agents’ subjective expected utilities.
From (16), we see that, conditional on observing \( s_R = g_j \), the government agency’s optimal policy depends on the sign of

\[
H_j \equiv v \left( \hat{\mu}_{hj} \ln \frac{C^d_{3hj}}{C^d_{3h-j}} + (1 - \hat{\mu}_{hj}) \ln \frac{C^d_{3lj}}{C^d_{3l-j}} \right) + (1 - v) \left( \hat{\mu}_{hj} \ln \frac{C^d_{3hj}}{C^d_{3h-j}} + (1 - \hat{\mu}_{hj}) \ln \frac{C^d_{3lj}}{C^d_{3l-j}} \right)
\]

\[
\hat{\mu}_{hj} \left( \ln \frac{v\hat{\mu}^d_{h-j}}{v\hat{\mu}^d_{hj}} + (1 - v)\hat{\mu}_{h-j} + v \ln \frac{\hat{\mu}^d_{hj}}{\hat{\mu}_{h-j}} + (1 - v) \ln \frac{\hat{\mu}_{hj}}{\hat{\mu}_{h-j}} \right) + (1 - v) \left( \ln \frac{v(1 - \hat{\mu}^d_{h-j}) + (1 - v)(1 - \hat{\mu}_{h-j})}{v(1 - \hat{\mu}^d_{hj}) + (1 - v)(1 - \hat{\mu}_{hj})} + v \ln \frac{1 - \hat{\mu}^d_{hj}}{1 - \hat{\mu}_{h-j}} + (1 - v) \ln \frac{1 - \hat{\mu}_{hj}}{1 - \hat{\mu}_{h-j}} \right)
\]

(19)

If (19) is positive, then social welfare is increasing in the probability of a revealing signal, and the government agency’s optimal policy is to forward its private signal with probability one \((\theta^*_j = 1)\). If the above expression is negative, the optimal policy is to send a signal that is the opposite of its private signal with probability one \((\theta^*_j = 0)\). Finally, if the above expression is zero, then the optimal solution is \(\theta^*_j \in [0, 1]\). Simple numerical tests show that the sets of parameters generating positive, negative and zero values on \(H_j\) are all nonempty. In particular, if agents believe that there is always a 50% probability that the government agency announces a high growth rate, regardless of its private signal, then their beliefs are not affected by the public signal. That is, in this case, the government agency is unable to affect social welfare through its public signal. Thus, there are infinitely many equilibria.

**Proposition 2.** If agents hold beliefs \(\hat{\theta}_h = \hat{\theta}_l = \frac{1}{2}\), then the government agency is unable to affect ex-post social welfare through its public signal. Thus, such beliefs support infinitely many equilibria in which \(\theta^*_j \in [0, 1]\) \((j = h, l)\).
Depending on the parameter values, the equilibria are either such that the government agency’s strategy is unique or such that there are infinitely many optimal strategies for the government agency. That is, in contrast to the equilibria we consider in the above proposition, there are other equilibria in which the government agency’s optimal strategy is unique. We call such equilibria unique-strategy equilibria. Since the agents’ beliefs need to be consistent with the set of optimal government agency strategies, its optimal strategy must coincide with the agents’ beliefs in a unique-strategy equilibrium.

**Lemma 2.** In a unique-strategy equilibrium, $\theta^*$ coincides with $\hat{\theta}$.

The result follows directly from item v of the equilibrium definition (Definition 1).

Since the expression in (19) does not depend on the government agency’s strategy, we can conclude that, in unique-strategy equilibria, the government agency’s strategy must be a “bang-bang” solution, which leaves us with four possible combinations: $(1, 1)$, $(0, 0)$, $(1, 0)$ and $(0, 1)$. Out of these strategies, $(1, 1)$ and $(0, 0)$ both reveal the government agency’s private signal to the public (the latter by always sending a public signal that is the opposite of the private one). It therefore seems natural that if the agents’ beliefs coincide with these agency strategies, the strategies will give rise to the same ex-post social welfare. As the following proposition shows, this is indeed the case.

**Proposition 3.** Given that $\hat{\theta}$ coincides with $\theta$, the strategies $\theta = (1, 1)$ ($\hat{\theta} = (1, 1)$) and $\theta = (0, 0)$ ($\hat{\theta} = (0, 0)$) result in the same ex-post social welfare.

If the government agency selects $(1, 0)$ and the agents’ beliefs coincide with this strategy—i.e., if the government agency always says “high growth” regardless of its private
signal—then, provided that the government agency sticks to this strategy, its public announcement is not valuable and the agents stick to their priors.\footnote{The imprecision of the government agency’s private signal makes this type of equilibrium possible. In the case when $\xi = 1$, so that the private signal fully reveals the true growth rate, this type of equilibrium is not possible (see the section “Benchmark case with $\xi = 1$” in the appendix). The explanation is that, in this case, the scenario in which the true growth rate is high given a low signal carries a weight of zero in the government agency’s objective function. This means that it is not optimal for the government agency to lie and say “high growth” when its private signal shows that the future growth will be low.} This is not the same as saying that they do not pay attention to the government agency’s announcement, because they would still update their beliefs if the government agency were to deviate from this strategy. However, in the case that we consider above, in which the agents believe that there is always a 50% probability of the government agency announcing a high growth rate, the agents do not pay attention to the government agency’s public announcement. We show below that these two situations give rise to the same ex-post social welfare.

**Proposition 4.** The strategy $\theta = (1, 0)$ in combination with beliefs $\widehat{\theta} = (1, 0)$ results in the same ex-post social welfare as the strategies $\theta = (s, t) \in [0, 1] \times [0, 1]$ in combination with beliefs $\widehat{\theta} = (\frac{1}{2}, \frac{1}{2})$.

One would think that the informative unique-strategy equilibria where $\theta^* = (1, 1)$ or $\theta^* = (0, 0)$ would dominate the unique-strategy equilibrium where the government agency always announces a high growth ($\theta^* = (1, 0)$). However, as Figures 1–4 show, the unique-strategy equilibria where $\theta^* = (1, 1)$ or $\theta^* = (0, 0)$ do not always result in an ex-post social welfare that is higher than the unique-strategy equilibrium in which $\theta^* = (1, 0)$. In fact,
the opposite can be true. The following proposition summarizes this finding.\textsuperscript{12}

**Proposition 5.** The unique-strategy equilibrium in which \( \theta^* = (1, 0) \) can result in a higher ex-post social welfare than the informative unique-strategy equilibria in which \( \theta^* = (1, 1) \) or \( \theta^* = (0, 0) \). Sufficient conditions are \( H_h | \hat{\theta}=(1,1) > 0, H_h | \hat{\theta}=(0,0) < 0, H_h | \hat{\theta}=(1,0) > 0, H_l | \hat{\theta}=(1,1) > 0, H_l | \hat{\theta}=(0,0) < 0, H_l | \hat{\theta}=(1,0) < 0 \) and \( W_l | \theta_l=1, \hat{\theta}=(1,1) = W_l | \theta_l=0, \hat{\theta}=(0,0) < W_l | \theta_l=0, \hat{\theta}=(1,0) \).

As seen in Figure 1b, given that the agents believe that the government agency forwards its private signal with probability one, the strategy of forwarding the private signal in the case in which it says “low growth” breaks down, provided that the fraction of dissenting agents is sufficiently large. Moreover, from Figure 3b, we see that—given the assumed parameter values—the strategy of always announcing a high growth can only be an equilibrium if the fraction of dissenting agents is sufficiently large. Our interpretation is that if the mass of dissenting agents is sufficiently large, the government agency’s objective function deviates sufficiently from the average agent in order for it not to forward valuable information.\textsuperscript{13} Table 1 also suggests that, if the dissenting agents’ prior differs “sufficiently” from the conforming agents’ prior, the only possible unique-strategy equilibrium is the uninformative one in which the government agency’s signal always says

\textsuperscript{12}Of course, a necessary condition for \( \theta^* = (1, 0) \) to achieve higher \textit{ex-post} social welfare than the informative unique-strategy equilibria is that the mass of dissenting agents be positive \( (v > 0) \). As mentioned earlier, if \( v = 0 \), then the government agency cannot affect \textit{ex-post} social welfare.

\textsuperscript{13}Recall that both the agency’s and the conforming agents’ prior probability of a high growth rate is \( p_h \), while the dissenting agents’ prior is \( p_h^d \neq p_h \).
“high growth.” A difference between the conforming and dissenting agents’ priors will of course also induce a difference between the government agency’s objective and that of the average agent. This is in line with the result that “the more similar agents’ preferences [sender and receiver], the more informative the equilibrium signal” (Crawford and Sobel, 1982, p. 1432). As one might suspect, the uninformative equilibrium in which the government agency always says “low growth” (0, 1) can be ruled out as a unique-strategy equilibrium because the government agency can increase ex-post social welfare by sending a high-growth public signal in the case that it receives a high-growth private signal.

4 Conclusion

This paper analyzes how a benevolent, privately-informed government agency would optimally convey information regarding future growth rates in a Lucas (1978) exchange-only economy where the growth rate can be either high or low. The government agency chooses between forwarding its private signal, which partially reveals the true future growth rate, and sending an opposite signal. We model two types of agent: conforming and dissenting. The conforming agents hold the same prior beliefs as the government agency, whereas the dissenting agents differ in their prior beliefs. In our model, both agents are rational Bayesian updaters, but our results readily extend to settings where a group of agents or all agents follow a behavioral updating rule. In deciding what public signal to send, the government agency seeks to maximize a standard ex-post measure of social welfare.

When both types of agent are present, the government agency is in some cases able
to affect the distribution of resources across the two future states (high and low growth). We identify both informative and uninformative equilibria. However, the informative equilibria do not always result in a higher *ex-post* social welfare than the uninformative equilibria. Key to this result is the information constraint: Without it, there would be no uninformative unique-strategy equilibria and the informative unique-strategy equilibria would dominate the uninformative equilibria in which there are infinitely many optimal strategies for the government agency (see the appendix). Some numerical examples suggest that the mass of dissenting agents or the difference in priors between the two types of agent needs to be sufficiently large in order for the uninformative equilibria to dominate.

With a larger mass of dissenting agents or a larger difference in priors, the government agency’s objective diverges more from that of the average agent. Hence, this result is in line with the conclusion that “the more similar agents’ preferences [sender and receiver], the more informative the equilibrium signal” (Crawford and Sobel, 1982, p. 1432).

We also analyze how the mass of the dissenting agents affects agents’ investment decisions and equilibrium asset prices. One result is that, if the dissenting agents have a more pessimistic (optimistic) prior regarding dividend growth than the conforming agents, then the larger the mass of the dissenting agents, the higher (lower) the bond price, or, equivalently, the lower (higher) the interest rate. The reason is that if the dissenting agents have a more pessimistic (optimistic) prior regarding dividend growth than the conforming agents, then the bond appears more (less) attractive to the dissenting agents.
Appendix

Agents’ equilibrium consumption

After having determined the agents’ posterior beliefs, we can consider the agents’ optimization problems. We can write the dissenting agents’ problem as

$$\max_{c_{2j}^d, c_{3hj}^d, c_{3lj}^d} u(c_{2j}^d) + \beta [\hat{\mu}_{hj}^d u(c_{3hj}^d) + (1 - \hat{\mu}_{hj}^d) u(c_{3lj}^d)]$$

s.t. $c_{2j}^d + q_{hj} c_{3hj}^d + q_{lj} c_{3lj}^d = D_2 + q_{hj} D_2 g_h + q_{lj} D_2 g_l$

$$c_{2j}^d \geq 0$$

$$c_{3hj}^d \geq 0$$

$$c_{3lj}^d \geq 0$$

Since $u'(c) \to +\infty$ as $c \to 0$, the last three constraints are not binding. Hence, the corresponding Lagrangian is

$$L_j^d = u(c_{2j}^d) + \beta [\hat{\mu}_{hj}^d u(c_{3hj}^d) + (1 - \hat{\mu}_{hj}^d) u(c_{3lj}^d)]$$

$$+ \lambda_j (D_2 + q_{hj} D_2 g_h + q_{lj} D_2 g_l - c_{2j}^d - q_{hj} c_{3hj}^d - q_{lj} c_{3lj}^d),$$

where $\lambda_j$ is the Lagrange multiplier. The first-order conditions with respect to initial and state-contingent consumption are

$$c_{2j}^d : u'(c_{2j}^d) - \lambda_j = 0,$$  \hspace{1cm} (22)

$$c_{3hj}^d : \beta \hat{\mu}_{hj}^d u'(c_{3hj}^d) - \lambda_j q_{hj} = 0,$$  \hspace{1cm} (23)

$$c_{3lj}^d : \beta (1 - \hat{\mu}_{hj}^d) u'(c_{3lj}^d) - \lambda_j q_{lj} = 0.$$  \hspace{1cm} (24)
Assuming constant relative risk aversion, so that \( u'(c) = c^{-\gamma} \), we have

\[
\begin{align*}
C_{2j}^d &= \lambda_j^{\frac{1}{\gamma}}, \\
C_{3hj}^d &= \left( \frac{\lambda_j q_{hj}}{\beta \hat{\mu}_{hj}^d} \right)^{\frac{1}{\gamma}}, \\
C_{3lj}^d &= \left( \frac{\lambda_j q_{lj}}{\beta (1 - \hat{\mu}_{hj}^d)} \right)^{\frac{1}{\gamma}}.
\end{align*}
\]

From the budget constraint, it follows that

\[
\lambda_j^{\frac{1}{\gamma}} = \frac{D_j (1 + q_{lj} g_{h} + q_{lj} g_{l})}{1 + \beta^\frac{1}{\gamma} \left[ (\hat{\mu}_{hj}^d)^{\frac{1}{\gamma}} q_{hj} + (1 - \hat{\mu}_{hj}^d)^{\frac{1}{\gamma}} q_{lj} \right]}
\]

We can use similar elementary lines to determine the optimal initial and state-contingent consumption of the conforming agents. With those findings, we can now move to analyzing the government’s optimal signal.

In order to achieve tractability, we solely focus on logarithmic utility functions \( \gamma = 1 \).\textsuperscript{14} In this case, the initial and state-contingent consumption of the dissenting agents is

\[
\begin{align*}
C_{2j}^d &= \frac{D_j (1 + q_{lj} g_{h} + q_{lj} g_{l})}{1 + \beta}, \\
C_{3hj}^d &= \frac{\beta \hat{\mu}_{hj}^d D_j (1 + q_{lj} g_{h} + q_{lj} g_{l})}{q_{hj} (1 + \beta)}, \\
C_{3lj}^d &= \frac{\beta (1 - \hat{\mu}_{hj}^d) D_j (1 + q_{lj} g_{h} + q_{lj} g_{l})}{q_{lj} (1 + \beta)}.
\end{align*}
\]

\textsuperscript{14}We get qualitatively similar results for general risk aversion, \( \gamma \). However, the case when \( \gamma \neq 1 \) is less tractable and we immediately need to resort to numerical solutions (see the section “General risk aversion” below).
Similarly, the consumption of the conforming agents is given by

\[ C_{2j} = \frac{D_2(1 + q_{hj}g_h + q_{lj}g_l)}{1 + \beta}, \]  

(32)

\[ C_{3hj} = \frac{\beta \tilde{\mu}_{hj} D_2(1 + q_{hj}g_h + q_{lj}g_l)}{q_{hj}} \frac{1}{1 + \beta}, \]  

(33)

\[ C_{3lj} = \frac{\beta (1 - \tilde{\mu}_{hj}) D_2(1 + q_{hj}g_h + q_{lj}g_l)}{q_{lj}} \frac{1}{1 + \beta}. \]  

(34)

The market-clearing conditions can be summarized as

\[ vC_{2j}^d + (1 - v)C_{2j} = D_2, \]  

(35)

\[ vC_{3hj}^d + (1 - v)C_{3hj} = D_2g_h, \]  

(36)

\[ vC_{3lj}^d + (1 - v)C_{3lj} = D_2g_l, \]  

(37)

where, by Walras’ law, market clearing in any two of these markets implies market clearing also in the third.

The first market-clearing condition in (35) can be rewritten as \( q_{hj}g_h + q_{lj}g_l = \beta \). Thus, if we combine this with the second market-clearing condition in (36), we obtain the prices of the AD securities:

\[ q_{hj} = \beta \left[ v\tilde{\mu}_{hj}^d + (1 - v)\tilde{\mu}_{hj} \right] g_h \]  

(38)

and

\[ q_{lj} = \beta \left[ v(1 - \tilde{\mu}_{hj}^d) + (1 - v)(1 - \tilde{\mu}_{hj}) \right] g_l. \]  

(39)

Here, we note that a stronger belief in a high growth rate among both agents leads to a higher price of consumption in the high-growth state and a lower price of consumption in the low-growth state.
The equilibrium consumption of the dissenting agents is given by

\[ C_{2j}^d = D_2, \]  
\[ C_{3hj}^d = \frac{\hat{\mu}_{hj}^d}{v\hat{\mu}_{hj}^d + (1 - v)\hat{\mu}_{hj}} D_2 g_h, \]  
\[ C_{3lj}^d = \frac{(1 - \hat{\mu}_{hj}^d)}{v(1 - \hat{\mu}_{hj}^d) + (1 - v)(1 - \hat{\mu}_{hj})} D_2 g_l, \]

and, similarly, the conforming agents’ equilibrium consumption is

\[ C_{2j} = D_2, \]  
\[ C_{3hj} = \frac{\hat{\mu}_{hj}}{v\hat{\mu}_{hj} + (1 - v)\hat{\mu}_{hj}} D_2 g_h, \]  
\[ C_{3lj} = \frac{(1 - \hat{\mu}_{hj})}{v(1 - \hat{\mu}_{hj}) + (1 - v)(1 - \hat{\mu}_{hj})} D_2 g_l. \]

**Benchmark case with \( \xi = 1 \)**

Here, we will consider the case when \( \xi = 1 \) so that the government agency’s signal, \( s_R \), fully reveals the future growth rate. In this case, one market of AD securities may fail to clear if, for example, agents think that the government agency forwards the revealing signal, \( s_R \), with probability one. That is, if, for example, the government agency’s signal says “high growth” and agents think that the government agency forwards its signal with probability one, then the state opposite of what is being signaled by the government agency will face a demand of zero, while there is a strictly positive supply of AD securities for that state. Hence, the market for AD securities for that state will fail to clear. Nevertheless, we can apply our equilibrium definition (Definition 1) to the market for the AD security that corresponds to the state signaled by the government agency. Likewise, since the
probability of the state opposite of that being signaled is zero, we can still consider *ex-post* social welfare.

Suppose now that $s_R$ is fully revealing ($\xi = 1$). Then the agents’ posterior beliefs are

\[
\hat{\mu}_{hh} = \frac{p_h \hat{\theta}_h}{p_h \hat{\theta}_h + (1 - p_h) \left(1 - \hat{\theta}_h\right)}
\]

(46)

\[
\hat{\mu}_{hl} = \frac{p_h \left(1 - \hat{\theta}_h\right)}{p_h \left(1 - \hat{\theta}_h\right) + (1 - p_h) \hat{\theta}_l}
\]

(47)

\[
\hat{\mu}_{dh} = \frac{p_d^h \hat{\theta}_h}{p_d^h \hat{\theta}_h + (1 - p_d^h) \left(1 - \hat{\theta}_h\right)}
\]

(48)

\[
\hat{\mu}_{dl} = \frac{p_d^l \left(1 - \hat{\theta}_h\right)}{p_d^l \left(1 - \hat{\theta}_h\right) + (1 - p_d^l) \hat{\theta}_l}.
\]

(49)

First, we will investigate whether there can exist a informative equilibrium where everyone believes in the government agency and the government agency forwards the private signal. In this case, the problem in (20) would reduce to a problem under certainty. If, under these beliefs, the government agency sends $s = g_h$, all agents would think that the high-growth state occurs with probability one and no agent would want to hold the claim to consumption in the low-growth state. Thus, the market for the claim to consumption in the low-growth state would break down and the consumption in the other state and the current consumption would look as follows in equilibrium.

\[
C_{2h}^d = C_{2h} = D_2
\]

(50)

\[
C_{3hh}^d = C_{3hh} = D_2 g_h
\]

(51)

Conversely, if the government agency sends $s = g_l$, the claim to consumption in the high-growth state would break down and the consumption in the other state and the
current consumption would look as follows in equilibrium.

\[
C_{2l}^d = C_{2l} = D_2
\]  
\[
C_{3ll}^d = C_{3ll} = D_2g_l
\]

Suppose \( s_R = g_l \). Then, sending \( s = g_h \) would result in a social welfare that is “negative infinity.” 15 A similar argument can be made in the case that \( s_R = g_h \). Thus, it is optimal for the government agency to forward its private signal: We have identified an equilibrium where the government agency reveals the true growth rate and everyone believes in the government agency.

Second, we will investigate whether there can exist a uninformative equilibrium where no one believes in the government agency and the government agency always sends a high-growth signal. If the agents’ beliefs are \( \hat{\theta} = (1, 0) \), then the agents’ equilibrium consumptions are given by the expressions in (29)–(34), where

\[
\hat{\mu}_{hh}^d = p_h
\]  
\[
\hat{\mu}_{hh} = p_h
\]  
\[
\hat{\mu}_{hl}^d = 0
\]  
\[
\hat{\mu}_{hl} = 0.
\]

Suppose that the government agency receives the signal \( s_R = g_l \). Then, its belief is

15Strictly speaking, because \( \ln c \) is not defined at \( c = 0 \), one would need to consider the limit of \( \frac{c^{1-\gamma} - 1}{1-\gamma} \) as \( \gamma \) approaches one from below.
\( \hat{\mu}_{hl} = 0 \), meaning that the expression for \( H_1 \) in (19) is given by

\[
H_1 = \ln\left[ v(1 - p_h^d) + (1 - v)(1 - p_h) \right] - v \ln(1 - p_h^d) - (1 - v) \ln(1 - p_h) > 0, \tag{58}
\]

where the inequality follows from the strict concavity of the logarithmic function and the circumstance that \( v \in (0, 1) \) and \( p_h^d \neq p_h \). Hence, this type of equilibrium does not exist when the government agency has perfect information regarding the growth rate.

Third, we consider the uninformative case when \( \hat{\theta} = (\frac{1}{2}, \frac{1}{2}) \). In this case, agents stick to their priors, regardless of what the government agency signals, so the government agency cannot affect agents’ consumption through its signaling. Therefore, there is an uninformative equilibrium in which \( \hat{\theta} = (\frac{1}{2}, \frac{1}{2}) \) and \( \theta^* \in [0, 1] \times [0, 1] \). However, we can conclude that this equilibrium yields a lower \textit{ex-post} social welfare than an informative one (cf. the first case above).

**Proof of Lemma 1**

By the no-arbitrage assumption, the prices of the stock\(^{16}\) and the bond are given by

\[
S_j = D_2(q_{hj}g_h + q_{lj}g_l) = \beta D_2 \tag{59}
\]

\[
B_j = q_{hj} + q_{lj} = \beta \left[ \frac{\hat{\mu}_{hj} - v(\hat{\mu}_{hj} - \hat{\mu}_{hj}^d)}{g_h} + \frac{1 - \hat{\mu}_{hj} + v(\hat{\mu}_{hj} - \hat{\mu}_{hj}^d)}{g_l} \right]. \tag{60}
\]

Here, we see that the stock price does not depend on beliefs. Hence, the government agency’s signal does not affect the price of the risky asset.

\(^{16}\)Note that the stock price is the price of a stock that has been stripped of its initial dividend.
Proof of Proposition 1

By Bayes’ Theorem, we have that

\[
\hat{\Pr}(g = g_h \mid s = g_h) = \frac{\hat{\Pr}(g = g_h \cap s = g_h)}{\hat{\Pr}(s = g_h)} = \frac{\hat{\Pr}(g = g_h)\hat{\Pr}(s = g_h \mid g = g_h)}{\hat{\Pr}(s = g_h)}.
\] (61)

Further, by the law of total probability,

\[
\hat{\Pr}(s = g_h) = \hat{\Pr}(s_R = g_h)\hat{\Pr}(s = g_h \mid s_R = g_h) + \hat{\Pr}(s_R = g_l)\hat{\Pr}(s = g_h \mid s_R = g_l),
\] (62)

and

\[
\hat{\Pr}(s = g_h \mid g = g_h) = \hat{\Pr}(s_R = g_h)\hat{\Pr}(s = g_h \mid g = g_h, s_R = g_h)
\]
\[+ \hat{\Pr}(s_R = g_l \mid g = g_h)\hat{\Pr}(s = g_h \mid g = g_h, s_R = g_l).
\] (63)

The government agency does not know the true growth rate, it only has access to the partially revealing private signal, \(s_R\). Thus, conditioning \(s\) on \(s_R\) and \(g\) is equivalent to conditioning \(s\) on \(s_R\) only, and we have that

\[
\hat{\Pr}(s = g_h \mid g = g_h) = \hat{\Pr}(s_R = g_h)\hat{\Pr}(s = g_h \mid s_R = g_h)
\]
\[+ \hat{\Pr}(s_R = g_l \mid g = g_h)\hat{\Pr}(s = g_h \mid s_R = g_l).
\] (64)

This implies that

\[
\hat{\Pr}(s = g_h \mid g = g_h) = \xi \hat{\theta}_h + (1 - \xi)(1 - \hat{\theta}_l).
\] (65)

Again, by the law of total probability,

\[
\hat{\Pr}(s_R = g_h) = \hat{\Pr}(g = g_h)\hat{\Pr}(s_R = g_h \mid g = g_h) + \hat{\Pr}(g = g_l)\hat{\Pr}(s_R = g_h \mid g = g_l).
\] (66)
\[
\hat{\Pr}(s_R = g_l) = \hat{\Pr}(g = g_h) \hat{\Pr}(s_R = g_l \mid g = g_h) + \hat{\Pr}(g = g_l) \hat{\Pr}(s_R = g_l \mid g = g_l). \tag{67}
\]

From (62), we have that
\[
\hat{\Pr}(s = g_h) = (p_h \xi + (1 - p_h)(1 - \xi))\hat{\theta}_h + (p_h(1 - \xi) + (1 - p_h)\xi)(1 - \hat{\theta}_l)
\]
\[
= p_h(\xi\hat{\theta}_h + (1 - \xi)(1 - \hat{\theta}_l)) + (1 - p_h)((1 - \xi)\hat{\theta}_h + \xi(1 - \hat{\theta}_l)). \tag{68}
\]

Thus, one of the conditional probabilities we seek is given by
\[
\hat{\Pr}(g = g_h \mid s = g_h) = \frac{p_h[\xi\hat{\theta}_h + (1 - \xi)(1 - \hat{\theta}_l)]}{p_h[\xi\hat{\theta}_h + (1 - \xi)(1 - \hat{\theta}_l)] + (1 - p_h)[(1 - \xi)\hat{\theta}_h + \xi(1 - \hat{\theta}_l)]}. \tag{69}
\]

The conditional probability \(\hat{\Pr}(g = g_h \mid s = g_l)\) can be calculated using the same steps as outlined above. This results in
\[
\hat{\Pr}(g = g_h \mid s = g_l) = \frac{p_h[\xi(1 - \hat{\theta}_h) + (1 - \xi)\hat{\theta}_l]}{p_h[\xi(1 - \hat{\theta}_h) + (1 - \xi)\hat{\theta}_l] + (1 - p_h)[(1 - \xi)(1 - \hat{\theta}_h) + \xi\hat{\theta}_l]}. \tag{70}
\]

In order to get to the dissenting agents’ conditional probabilities, one just needs to take into account that they have a different prior, \(p^d_h\).

**Proof of Proposition 2**

If \(\hat{\theta}_h = \hat{\theta}_l = \frac{1}{2}\), it follows from equations (5) through (8) that \(\hat{\mu}_h^d = \hat{\mu}_l^d = p^d_h\) and \(\hat{\mu}_h = \hat{\mu}_l = p_h\). Thus, from (19), we see that \(H_j = 0\), so that \(\theta^*_j \in [0,1] \quad (j = h,l)\). Now we can go through the equilibrium definition (Definition 1) and confirm that \(\theta^*_j \in [0,1] \quad (j = h,l)\) holds in equilibrium.
Proof of Proposition 3

By substituting \( \hat{\theta} = (1, 1) \) and \( \hat{\theta} = (0, 0) \), respectively, into the posterior beliefs in (5) through (8), we see that

\[
W_j|_{\theta_j=1, \hat{\theta}=(1,1)} = W_j|_{\theta_j=0, \hat{\theta}=(0,0)} \quad (j = h, l). \tag{71}
\]

Proof of Proposition 4

If \( \hat{\theta} = (1, 0) \), the posterior beliefs conditional on observing a high-growth public signal are \( \hat{\mu}_{hh} = p_h \) and \( \hat{\mu}_{dh}^d = p_d^d \), respectively. Further, if \( \hat{\theta} = (\frac{1}{2}, \frac{1}{2}) \), then the posterior beliefs are \( \hat{\mu}_{hh} = \hat{\mu}_{hl} = p_h \) and \( \hat{\mu}_{dh}^d = \hat{\mu}_{dl}^d = p_h^d \), respectively. Thus, we have that

\[
W_j|_{\theta=(1,0), \hat{\theta}=(1,0)} = W_j|_{\theta=(s,t)\in[0,1]\times[0,1], \hat{\theta}=(\frac{1}{2}, \frac{1}{2})} \quad (j = h, l). \tag{72}
\]

Proof of Proposition 5

Consider the following parameter values:

- \( v = 0.5, \ p_h^d = 0.60, \ p_h = 0.80, \ D_2 = 1, \ g_h = 1.2, \ g_l = 0.8, \ \xi = 0.70, \ \beta = 0.95. \)

Under these parameter values, we have that \( H_h|_{\hat{\theta}=(1,1)} > 0, \ H_h|_{\hat{\theta}=(0,0)} < 0, \ H_h|_{\hat{\theta}=(1,0)} > 0, \ H_l|_{\hat{\theta}=(1,1)} > 0, \ H_l|_{\hat{\theta}=(0,0)} < 0 \) and \( H_l|_{\hat{\theta}=(1,0)} < 0. \) Thus, the parameter values support all of the unique-strategy equilibria we mention in the proposition.

Now, given these parameter values, consider the \textit{ex-post} social welfare in the case of a
low-growth private signal:

\[ W_{l|\theta_l=1, \hat{\theta}=(1,1)} = W_{l|\theta_l=0, \hat{\theta}=(0,0)} = 3.35 \cdot 10^{-3} < 4.49 \cdot 10^{-3} = W_{l|\theta_l=0, \hat{\theta}=(1,0)}. \]  \hspace{1cm} (73)

Thus, under the parameter values given in the proposition, *ex-post* social welfare conditional on a low-growth private signal is higher in the unique-strategy equilibrium in which \( \theta^* = (1,0) \) than in the informative unique-strategy equilibria in which \( \theta^* = (1,1) \) and \( \theta^* = (0,0) \), respectively.

**Two\((n)\)-period case**

Here, we will explain how our results extend to the two\((n)\)-period case. The two-period case separates into two one-period problems, provided that growth rates and signals are IID over time. This then generalizes to the case of \( n \) periods. Key to this result is that according to the agents’ priors, all probability mass is at \( p_d^h \) and \( p_h \), respectively. A more general setting would have a prior distribution over the probability of a high growth rate (e.g., a beta distribution). In that case, observations in previous periods would affect the current beliefs regarding the distribution of the probability of a high growth rate. With just a couple of periods, the influence of the observations of signaling and growth realizations is likely to be small provided that the prior distribution is not too diffuse. Therefore, we think that our assumptions regarding the priors is a reasonable one in case there are not too many periods.

For example, consider the two-period setting. We can apply exactly the same analysis as we did previously to the last (second) period. Because the priors are concentrated
around \( p^d_h \) and \( p_h \), respectively, the government agency’s signal in the first period will not affect social welfare in the second period, and so the problem of choosing a signal to maximize social welfare over the two periods reduces to a one-period problem. Thus, the two-period case can be separated into two one-period cases.

**General risk aversion**

In this subsection, we show how our results extend to the case with general risk aversion—i.e., the case when \( \gamma \) is different from one. Then from (25)–(28), optimal consumption is given by

\[
C^d_{2j} = \frac{D_2(1 + q_{hj}g_h + q_{lj}g_l)}{1 + \beta^\frac{1}{\gamma} \left[ (\hat{\mu}^d_{hj})^{\frac{1}{\gamma}} q_{hj}^{1 - \frac{1}{\gamma}} + (1 - \hat{\mu}^d_{hj})^{\frac{1}{\gamma}} q_{lj}^{1 - \frac{1}{\gamma}} \right]} ,
\]

(74)

\[
C^d_{3hj} = \frac{D_2(1 + q_{hj}g_h + q_{lj}g_l)}{1 + \beta^\frac{1}{\gamma} \left[ (\hat{\mu}^d_{hj})^{\frac{1}{\gamma}} q_{hj}^{1 - \frac{1}{\gamma}} + (1 - \hat{\mu}^d_{hj})^{\frac{1}{\gamma}} q_{lj}^{1 - \frac{1}{\gamma}} \right]} \left( \frac{q_{hj}}{\beta \hat{\mu}^d_{hj}} \right)^{-\frac{1}{\gamma}} ,
\]

(75)

\[
C^d_{3lj} = \frac{D_2(1 + q_{hj}g_h + q_{lj}g_l)}{1 + \beta^\frac{1}{\gamma} \left[ (\hat{\mu}^d_{hj})^{\frac{1}{\gamma}} q_{hj}^{1 - \frac{1}{\gamma}} + (1 - \hat{\mu}^d_{hj})^{\frac{1}{\gamma}} q_{lj}^{1 - \frac{1}{\gamma}} \right]} \left( \frac{q_{lj}}{\beta (1 - \hat{\mu}^d_{hj})} \right)^{-\frac{1}{\gamma}} ,
\]

(76)

\[
C_{2j} = \frac{D_2(1 + q_{hj}g_h + q_{lj}g_l)}{1 + \beta^\frac{1}{\gamma} \left[ (\hat{\mu}_{hj})^{\frac{1}{\gamma}} q_{hj}^{1 - \frac{1}{\gamma}} + (1 - \hat{\mu}_{hj})^{\frac{1}{\gamma}} q_{lj}^{1 - \frac{1}{\gamma}} \right]} ,
\]

(77)

\[
C_{3hj} = \frac{D_2(1 + q_{hj}g_h + q_{lj}g_l)}{1 + \beta^\frac{1}{\gamma} \left[ (\hat{\mu}_{hj})^{\frac{1}{\gamma}} q_{hj}^{1 - \frac{1}{\gamma}} + (1 - \hat{\mu}_{hj})^{\frac{1}{\gamma}} q_{lj}^{1 - \frac{1}{\gamma}} \right]} \left( \frac{q_{hj}}{\beta \hat{\mu}_{hj}} \right)^{-\frac{1}{\gamma}} ,
\]

(78)

\[
C_{3lj} = \frac{D_2(1 + q_{hj}g_h + q_{lj}g_l)}{1 + \beta^\frac{1}{\gamma} \left[ (\hat{\mu}_{hj})^{\frac{1}{\gamma}} q_{hj}^{1 - \frac{1}{\gamma}} + (1 - \hat{\mu}_{hj})^{\frac{1}{\gamma}} q_{lj}^{1 - \frac{1}{\gamma}} \right]} \left( \frac{q_{lj}}{\beta (1 - \hat{\mu}_{hj})} \right)^{-\frac{1}{\gamma}} .
\]

(79)

From two of the market-clearing conditions in (35)–(37), we can solve for the prices of
the AD securities numerically; thus, we can determine the equilibrium level of consumption and *ex-post* social welfare. We pick equations (35) and (37), and we set relative risk aversion equal to two \((\gamma = 2)\).

The social welfare function is given by

\[
W_j = \theta_j \left\{ v \left( \frac{(C^d_{2j})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} + \beta \left[ \frac{\hat{\mu}_{hj}^G (C^d_{3hj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} + (1 - \hat{\mu}_{hj}^G) \frac{(C^d_{3lj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} \right] \right) 
+ (1 - v) \left[ \frac{C^d_{2j}}{1-\gamma} + \beta \left[ \frac{\hat{\mu}_{hj}^G (C^d_{3hj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} + (1 - \hat{\mu}_{hj}^G) \frac{(C^d_{3lj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} \right] \right] 
+ (1 - \theta_j) \left\{ v \left( \frac{C^d_{2j}}{1-\gamma} + \beta \left[ \frac{\hat{\mu}_{hj}^G (C^d_{3hj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} + (1 - \hat{\mu}_{hj}^G) \frac{(C^d_{3lj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} \right] \right) 
+ (1 - v) \left[ \frac{C^d_{2j}}{1-\gamma} + \beta \left[ \frac{\hat{\mu}_{hj}^G (C^d_{3hj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} + (1 - \hat{\mu}_{hj}^G) \frac{(C^d_{3lj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} \right] \right] \right\} 
\right\}
\] 
(80)

Thus, conditional on observing \(s_R = g_j\), the government agency’s optimal policy depends on the sign of

\[
H_j \equiv v \left[ \frac{\hat{\mu}_{hj}^G (C^d_{3hj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} - \frac{(C^d_{3hj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} \right] + (1 - \hat{\mu}_{hj}^G) \left[ \frac{(C^d_{3lj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} - \frac{(C^d_{3lj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} \right] 
+ (1 - v) \left[ \frac{C^d_{3hj}}{1-\gamma} - \frac{(C^d_{3hj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} \right] + (1 - \hat{\mu}_{hj}^G) \left[ \frac{C^d_{3lj}}{1-\gamma} - \frac{(C^d_{3lj})^{\frac{1-\gamma}{1-\gamma}}}{1-\gamma} \right] 
\] 
(81)

Our numerical calculations show that our main results remain the same also for a nonlogarithmic power utility. From Table 2, we can see that there can be both informative and uninformative equilibria. Further, the uninformative equilibria can dominate the informative ones in terms of *ex-post* social welfare.\(^{17}\)

\(^{17}\)For example, with \(\gamma = 2\), \(p^d_h = 0.60\), \(p_h = 0.80\), \(\xi = 0.60\), and \(v = 0.5\), social welfare in the case of a low-growth signal \((W_j)\) is higher in the uninformative equilibrium, in which \(\theta^* = (1, 0)\), than in the informative equilibrium, in which \(\theta^* = (1, 1)\): The values are \(-1.86\) and \(-1.91\), respectively.
Alternative welfare function

Here, we consider an alternative welfare function (the average ex-ante expected utility of the agents) and show that our main results continue to hold. Instead of maximizing ex-post social welfare, the government agency now seeks to maximize

\[ W_a^j = E^P \left[ \int_{[0,1]} E^P \left[ u(C_2^j) + \beta u(C_3^j) \mid s \right] \, ds \right] \]  

by choosing the probability \( \theta_j \) by which it forwards its private signal \( s_R = g_j \).

Suppose now that the government agency receives a low-growth signal, \( s_R = g_l \). Then, ex-ante social welfare is

\[ W_a^l = \theta_l \{ v \left[ \hat{\mu}_{hl} \ln C_{3hl} + (1 - \hat{\mu}_{hl}) \ln C_{3ll} \right] \\
+ (1 - v) \left[ \ln C_{2l} + \beta (\hat{\mu}_{hl} \ln C_{3hl} + (1 - \hat{\mu}_{hl}) \ln C_{3ll}) \right] \}
+ (1 - \theta_l) \{ v \left[ \ln C_{2h} + \beta (\hat{\mu}_{hh} \ln C_{3hh} + (1 - \hat{\mu}_{hh}) \ln C_{3lh}) \right] \\
+ (1 - v) \left[ \ln C_{2hl} + \beta (\hat{\mu}_{hl} \ln C_{3hl} + (1 - \hat{\mu}_{hl}) \ln C_{3lh}) \right] \}, \]

and thus, the optimal strategy, \( \theta_l^* \), depends on the sign of

\[ H_l = v \left[ \hat{\mu}_{hl} \ln C_{3hl} + (1 - \hat{\mu}_{hl}) \ln C_{3ll} - \hat{\mu}_{hh} \ln C_{3hh} + (1 - \hat{\mu}_{hh}) \ln C_{3lh} \right] \\
+ (1 - v) \left[ \hat{\mu}_{hl} \ln C_{3hl} + (1 - \hat{\mu}_{hl}) \ln C_{3ll} - \hat{\mu}_{hh} \ln C_{3hh} + (1 - \hat{\mu}_{hh}) \ln C_{3lh} \right]. \]

As seen in Figures 5 and 6, we can obtain informative as well as uninformative equilibria. Moreover, the uninformative equilibria can also dominate the informative ones under this alternative welfare function.\(^\text{18}\)

\(^{18}\)For example, if \( p_d^d = 0.55, p_h = 0.95, \xi = 0.70 \) and \( v = 0.5 \), then \( W_1^a \mid \theta^* = \hat{\theta} = (1, 0) = 0.190 > 0.188 = W_1^a \mid \theta^* = \hat{\theta} = (1, 1) \).
References


Table 1: The possible unique-strategy equilibria for the government agency \((\theta^\ast)\) for various values on the dissenting agents’ prior \((\theta^d)\) when the fraction of dissenting agents \((v)\) is 30% and 60%, respectively. We have assumed that \(p_h = 0.80\) and \(\xi = 0.70\). Note that the government agency’s strategy depends on the agents’ beliefs regarding the likelihood that the government agency forwards its private signal \((\hat{\theta})\). In a unique-strategy equilibrium \(\theta^* = \hat{\theta}\).

<table>
<thead>
<tr>
<th>Dissenting agents’ prior</th>
<th>Possible unique-strategy equilibria</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>0.10</td>
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<td>(1, 1), (0, 0), (1, 0)</td>
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<td>(1, 1), (0, 0), (1, 0)</td>
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<td>(1, 1), (0, 0)</td>
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<tr>
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<td>(1, 1), (0, 0)</td>
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<tr>
<td>(v = 0.60)</td>
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<tr>
<td>0.10</td>
<td>(1, 0)</td>
</tr>
<tr>
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<td>(1, 1), (0, 0)</td>
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Table 2: \(H_h\) and \(H_l\) as a function of the fraction of dissenting agents \((v)\) for \(\hat{\theta} = (1, 1)\) and \(\hat{\theta} = (1, 0)\), respectively, when the coefficient of relative risk aversion equals two \((\gamma = 2)\). We have assumed the following parameter values: \(p^d_h = 0.60\), \(p_h = 0.80\) and \(\xi = 0.60\).

<table>
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<tr>
<th>(v)</th>
<th>(H_h)</th>
<th>(H_l)</th>
<th>(H_h)</th>
<th>(H_l)</th>
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</table>
Figures 1a and 1b: $H_h$ and $H_l$ in (19) as a function of the mass of the dissenting agents ($v$). A positive (negative) value means that social welfare is increasing (decreasing) in the probability that the government agency forwards its private signal. We have assumed the following parameter values: $p_d^h = 0.60$, $p_h = 0.80$, $\xi = 0.70$, and $\tilde{\theta} = (\tilde{\theta}_h, \tilde{\theta}_l) = (1, 1)$. 
Figures 2a and 2b: $H_h$ and $H_l$ in (19) as a function of the mass of the dissenting agents ($v$). A positive (negative) value means that social welfare is increasing (decreasing) in the probability that the government agency forwards its private signal. We have assumed the following parameter values: $p^d_h = 0.60$, $p_h = 0.80$, $\xi = 0.70$, and $\hat{\theta} = (\hat{\theta}_h, \hat{\theta}_l) = (0, 0)$. 

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Figures 3a and 3b: $H_h$ and $H_l$ in (19) as a function of the mass of the dissenting agents ($v$). A positive (negative) value means that social welfare is increasing (decreasing) in the probability that the government agency forwards its private signal. We have assumed the following parameter values: $p^d_h = 0.60$, $p_h = 0.80$, $\xi = 0.70$, and $\tilde{\theta} = (\tilde{\theta}_h, \tilde{\theta}_l) = (1, 0)$. 

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Figures 4a and 4b: The differences in *ex-post* social welfare in the cases where $s_R = g_h$ (top) and $s_R = g_l$ (bottom) between the strategies $\theta = (1,1)$ and $\theta = (1,0)$ as a function of the mass of the dissenting agents ($v$), provided that the agents beliefs are $\theta = (1,1) \text{ and } \hat{\theta} = (1,0)$, respectively. We have assumed the following parameter values: $p_{hl}^d = 0.60$, $p_h = 0.80$, $\xi = 0.70$, $D_2 = 1$, $g_h = 1.2$, $g_l = 0.8$, and $\beta = 0.95$. 

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Figure 5: $H_1$ in (84) as a function of the mass of the dissenting agents ($v$). A positive (negative) value means that social welfare is increasing (decreasing) in the probability that the government agency forwards its private signal. We have assumed the following parameter values: $p_h^0 = 0.55$, $p_h = 0.95$, $\xi = 0.70$, and $\hat{\theta} = (\hat{\theta}_h, \hat{\theta}_l) = (1, 1)$. 
Figure 6: $H_l$ in (84) as a function of the mass of the dissenting agents ($v$). A positive (negative) value means that social welfare is increasing (decreasing) in the probability that the government agency forwards its private signal. We have assumed the following parameter values: $p^d_h = 0.55$, $p_h = 0.95$, $\xi = 0.70$, and $\hat{\theta} = (\hat{\theta}_h, \hat{\theta}_l) = (1, 0)$. 

\[ H_l \]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{H in (84) as a function of the mass of the dissenting agents ($v$). A positive (negative) value means that social welfare is increasing (decreasing) in the probability that the government agency forwards its private signal.} 
\end{figure}
Growth Forecasts, Belief Manipulation and Capital Markets

FREDERIK LUNDTOFTE | PATRICK LEONI

We analyze how a benevolent, privately informed government agency would optimally release information about the economy’s growth rate when the agents hold heterogeneous beliefs. We model two types of agent: “conforming” and “dissenting.” The former has a prior that is identical to that of the government agency, whereas the latter has a prior that differs from that of the government agency. We identify both informative and uninformative equilibria and demonstrate that the uninformative equilibria can dominate the informative ones in terms of ex-post social welfare.

**Keywords:** Social welfare, information, forecasting, asset pricing, heterogeneous beliefs

**JEL codes:** D83, G11, G12